Creativity and Mathematics Education

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Abstract: A paper like this you can read twice. Therefore we first present our theoretical analysis and then some examples. For the oral presentation at the conference we will start with the examples and elaborate only a few aspects of "which are the mental processes to further creative thinking in mathematics education?": We must further individual and social abilities, we need challenging problems, and the nature of activities in the class room must integrate more spontaneous ideas and more (unconscious and intuitive) common-sense knowledge.

Key words: creativity, challenges, Vorstellungen, subjective domain of experiences, cognitive aspects, polarity in thinking, common sense

1. Problem Solving and Creativity

Already KIENEL (1977) distinguishes in his dissertation five categories of mathematical "problems". Problems of Types I – III can be solved by applying a rule or an algorithm or a procedure. In problems of Type I the rule or algorithm or procedure is mentioned explicitly, in problems of Type II the rule or algorithm or procedure is known to the problem solver, but it is not mentioned explicitly. The rules or algorithms or procedures necessary to solve a problem of Type III first must be constructed by the problem solver via combining known rules or algorithms or procedures. Problems of Type IV are given verbally as "a real world" problem, and the mathematical content first must be analyzed and transformed into a mathematical problem to get then a problem of Type I-III.

Type V puts together all those "real world" problems, where neither the knowledge of rules or algorithms or procedures nor a specific knowledge of facts, of data, of relations, of properties, etc. is sufficient to get a solution. Especially "open" problems or "challenges" are problems of Type V. To solve these problems we need a new idea or a "cognitive jump", "a divergent or creative thinking is necessary" (KIENEL, p. 122). This leads us to a central question.

2. What Does Creativity Mean?

How can we describe or define "creative thinking"? Many experts from different disciplines give various descriptions, but there is no standardized answer. "Creativity" is a highly complex phenomenon, and for some people it seems to be somehow incompatible with mathematics teaching. The traditional style of working in the mathematics classroom seems not to allow many creative ideas.

But again, how can we characterize "creative thinking"? We will not provoke the impression that creativity can be described by a long list of isolated items nor that such a list may help to identify or to develop creative ideas. But, as a first step to further the development of creativity, the flavor of such lists may help teachers and text book writers when they prepare classroom lessons. To develop and further creativity in mathematics education teachers and students need more than a correct and solid mathematical knowledge.

REN ZIZHAO (1999) demands *independence* ("instead of simply repeating other's old tricks") and *relative originality*. For KIESSWETTER (1983) *flexible thinking* is one of the most important abilities. For BISHOP (1981) two complementary modes of thinking are necessary, a logical, one-dimensional, language orientated aspect and a visual, more-dimensional, intuitive view. KRAUSE et al. (in ZIMMERMANN 1999, p. 129ff) even distinguish four basic components for creative thinking, *finding analogies*, *double representations* (visual-perceptual / formal-logical), *multiple classifications* and *reducing complexity*. When GRAY and TALL (1991) and others refer to "procepts" or "encapsulation of a process" they also describe abilities necessary for being creative. Again, there are various aspects, but there is no standardized description of "creativity".

A mathematics teaching which furthers creative thinking needs specific environments. In our research group in Muenster we try to concentrate on three aspects. First, we must further *individual and social components*, like motivation, curiosity, self-confidence, flexibility, engagement, humor, imagination, happiness, acceptance of oneself and others, satisfaction, success, ... We need a competitive atmosphere which still allows spontaneous actions and reactions, we need responsibility combined with voluntariness, ... We need tolerance and freedom, for the individuals to express their views, and within the group to further a fruitful communication. We need profound discussions as well as intuitive or spontaneous inputs and reactions.

Second, we need *"challenging problems"*. They must be fascinating, interesting, exciting, thrilling, important, provoking, ... Open ended problems are welcome or challenging problems with surprising contexts and results, ... We must connect the problems with the individual daily life experiences of the students, we must meet their fields of experiences and their interest areas. The students must be able to identify themselves with the problem and its possible solution(s).

And third, the children must develop *important abilities*. They must learn to explore and to structure a problem, to invent own or to modify given techniques, to listen and argue, to define goals, to cooperate in teams, ... We need children who are active, who discover and experience, who enjoy and have fun, who guess and test, who can laugh on own mistakes, ... That means, another step to further creative thinking is to further the development of these abilities. But they are demanding abilities and not simple skills. They rely and depend on a complex system of cognitive processes. Perhaps it may help a bit to analyze these internal processes.

3. Cognitive Aspects

A successful problem solving depends on the cognitive structure the problem solver has. An appropriate " internal representation" or concept image is necessary. The problem solver must have adequate " Vorstellungen" (MEISSNER 2002). These "Vorstellungen"[1] are like scripts or frames or micro worlds, and they are personal and individual. They are "Subjective Domains of Experiences"[2] (BAUERSFELD 1983).

The goal of mathematics education is to develop mathematical "Vorstellungen" which are extensive and effective, which are rich and flexible. We distinguish two kinds of "Vorstellungen", which we call "spontaneous Vorstellungen" and "reflective Vorstellungen". Thus we refer to a polarity in thinking which already was discussed before by many other authors[3].

"Reflective Vorstellungen" may be regarded as an internal mental copy of a net of knowledge, abilities, and skills, a net of facts, relations, properties, etc. where we have a conscious access to. Reflective Vorstellungen mainly are the result of a teaching. The development of "reflective Vorstellungen" certainly is in the center of mathematics education. Here a formal, logical, deterministic, and analytical thinking is the goal. To reflect and to make conscious are the important activities. We more or less do not realize or even ignore or suppress intuitive or spontaneous ideas. A traditional mathematics education does not emphasize unconsciously produced feelings or reactions. In mathematics education there is no space for informal pre-reflections, for an only "general" or "global" or "overall" view, or for uncontrolled spontaneous activities. Guess and test or trial and error are not considered to be a valuable mathematical behavior. But all these components are necessary to develop "spontaneous Vorstellungen". And these spontaneous Vorstellungen mainly develop unconsciously or intuitively.

Both types of " Vorstellungen" together form individual "Subjective Domains of Experiences" (SDE). For a well developed and powerful SDE both is essential, a sound and mainly intuitive "common-sense" and a conscious knowledge of rules and facts. Both aspects belong together like the two sides of a coin. And whenever necessary the individual must be able, often unconsciously, to switch from the one side to the other. But the way of looking at things is different. Spontaneous Vorstellungen and reflective Vorstellungen often interfere, positively or negatively. The view on facts, relations, or properties "suddenly" changes.

There also is a competition between different SDEs to become dominant when a new problem is presented. Intuitively, the then chosen SDE often remains dominant even when conflicts arise. The individual rather prefers to ignore the conflict than to modify the SDE or to adopt another SDE. And in mathematics education it is quite natural that a "analytical-logical" behavior remains dominant and that conflicting common-sense experiences or

[1] We will use the German word Vorstellungen instead of the vague expression " internal representation"
[2] " Subjektive Erfahrungsbereiche" in German
[3] e.g. VYGOTZKI talks about spontaneous and scientific concepts, GINSBURG compares informal work and written work, or STRAUSS discusses a common sense knowledge vs. a cultural knowledge. STRAUSS (1982) especially has pointed out that these two types of knowledge are quite different by nature, that they develop quite differently, and that sometimes they interfere and conflict (" U -shaped" behavior).

spontaneous ideas get ignored[4]. The chosen SDE even then remains dominant when the reflective Vorstellungen obviously are not sufficient to solve the problem. Then the related rules and procedures just get "reduced" or "simplified" or get replaced by easier "mechanisms".

Summarizing, we distinguish two types of internalizing "external representations"[5] and experiences. We get (mainly conscious) reflective Vorstellungen and (often intuitive) "spontaneous" Vorstellungen. Related to the momentarily situation both types together create or modify an individual "Subjective Domain of Experiences" in which this situation is imbedded then. But what does creativity mean in this context?

4. Again, What Does Creativity Mean?

Studying the diverse descriptions how to define "creativity" we can detect some common aspects. We need more than reflective Vorstellungen (not only "simply repeating other's old tricks"). Flexible thinking is demanded (KIESSWETTER), especially in two complementary modes (BISHOP, KRAUSE). And now we should add, we need flexible thinking with regard to the reflective Vorstellungen **and** with regard to the spontaneous Vorstellungen and we need a flexible thinking to combine or interweave or expand these two types of Vorstellungen.

Flexible thinking includes independence. We must not rely on a few dominant SDEs, we need the chance to experience, to construct, and to reflect many divergent SDEs. We then can detect and discuss analogies and differences and multiple classifications. Reflecting these activities we gain more insight and reduce the complexity. A social communication is the vehicle to widen consciously the horizon.

To further creative thinking in mathematics education we need more than powerful reflective Vorstellungen. Intuitive and spontaneous components are necessary. Each SDE is a mixture which allows different ways of looking at things. A balance between an reflective arguing and a common-sense thinking must become the goal in the class room: Let us start with a typical situation and try to collect and to discuss then all the individual SDEs coming up. Let us try not to separate artificially our daily life knowledge and experiences from the development of the "scientific concepts" in mathematics education. Above we had listed aspects which we think are necessary for a creative mathematics teaching:

- We must further individual and social abilities and
- we need challenging problems.

And now we can add:

- We need more spontaneous ideas and more (unconscious and intuitive) common sense knowledge.

Creative thinking then may develop as a powerful ability to interact between reflective and spontaneous Vorstellungen.

5. Challenges

There are many problems in mathematics education which are not "challenging" (KIENEL Types I – IV). But to further creative thinking in mathematics education we need really challenging problems. These challenges must activate or create not only reflective Vorstellungen but also related common-sense or spontaneous Vorstellungen. And then a powerful interaction between these two types of Vorstellungen should start to create solutions.

We will present some examples to further the development of spontaneous Vorstellungen, and examples where we think the process of finding solutions might help to further creative thinking[6]. And we will comment how our theoretical considerations above are related to each of these examples.

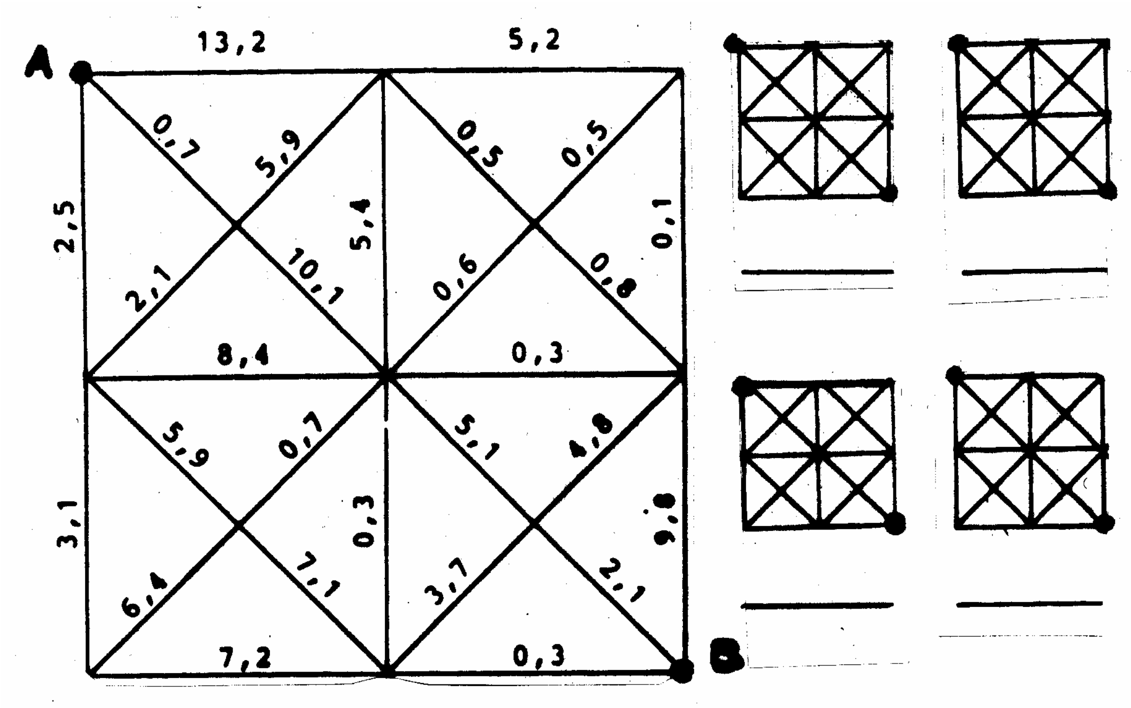
5.1. Decimal Grid (see Meissner e.a. 1999, p. 41)

Select a path from A to B. Change the direction at each crossing. Multiply (with a calculator) the numbers of each step you go. Find the path with the smallest product. You have 4 trials.

[4] There are interviews where children give different results to the „same" problem. For example adding " 50+25" in a "number world" may have a different result than adding the same numbers in a " money world". Those children often do not see a conflict, because unconsciously they just react in two different SDEs and " in mathematics it is different".

[5] " Darstellungen" in German, see MEISSNER (2002)

[6] more examples you may find in these proceedings or in SHEFFIELD (2003)



At a first glance, the problem is easy, let's start immediately: Take always the smallest number, write down your result. But then:

- How to find another better path?
- Are there rules to find the best path?

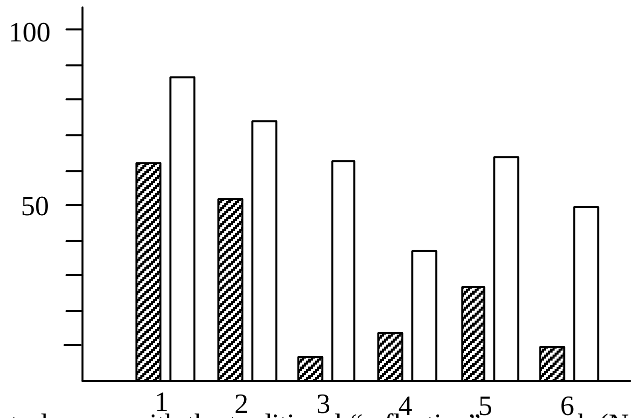
Cognitive jumps (SDEs get changed):

- Multiplication not always makes bigger
- More factors may give a smaller product
- Running in a circle forth and forth (i.e. …0.3 x 0.8 x 0.6 x …)
- infinite path (→ intuitive concept of limit)

5.2. Teaching Percentages (see VELIKOVA 2003, p. 34f)

There are calculators which work syntactically like we speak in our daily life:
"635 + 13 % = ..." needs the key stroke sequence

| 6 | | 3 | | 5 | + | 1 | | 3 | | % | = | . | | |



We taught percentages with the percent key, without using formulae or reverse functions or algebraic transformations of formulae. If necessary the missing values had to be guessed and verified by pressing always the same key stroke sequence from above. The students became excellent in guessing each value and they developed an astonishing "%-feeling". We administered the same test with 6 problems in our experimental group (white bars, N ≈ 250) and in a control group with the traditional "reflective" approach (N ≈ 500, dark bars).

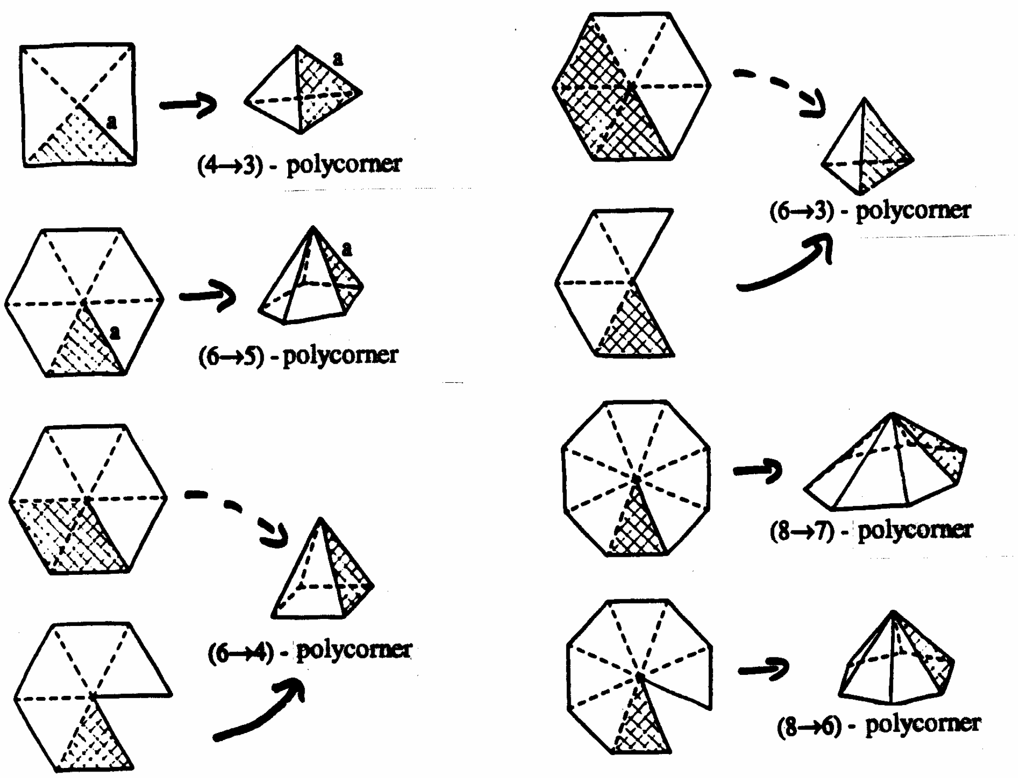
5.3. One-Way-Principle (see Meissner 2003)

Our method to use always the same key stroke sequence, eventually by guessing and testing the missing variables, we have called "One-Way-Principle" (OWP). The OWP is an intermediate step between simple examples with easy numbers and the algebraic generalization with formulae or functions and reverse functions and algebraic transformations. The OWP is a method to develop intuitive and spontaneous Vorstellungen about the relations between and about the order of magnitude of the many variables. Using guess and test develops an intuitive concept of "Percentages". The students spontaneously can guess quite exactly the result already before they start their computation.

A similar approach is possible to teach the topics "Interest", "Compound interest", "Growth and Decay", and others. We urge our students to write protocols from their guess and test work because these protocols are an excellent picture of their (mainly intuitive) *Vorstellungen.* Discussions then can bring the shift from an unconscious feeling to a conscious insight. We also used the OWP to teach the concept of "Functions". The traditional school curriculum has had not much success in developing a deeper understanding between the gestalt of a graph and the related algebraic term. Using guess and test with computers we developed that missing link for linear and quadratic functions: Our students very easily could sketch the gestalt for a given term and write down a term for a given graph.

5.4. Invent your own solids (see Meissner e.a. 1999, p. 177f)

We cut from a cardboard (about 250 g/m²) regular n-gons and stick them together to a variety of different "polycorners". Use paper clips to combine these "bricks". Most interesting are the (6→n)-polycorners (for n = 5, 4, or 3) because of their equilateral triangles.



______ cutting ------ folding xxxxxx overlapping

5.5. Imagine Solids

Given is a solid where the base and the upper face are parallel and congruent. Describe the solid. Base and upper face are regular n-polygons, describe again. Assume the solid is not a prism, but base and upper face still are regular n-polygons, describe. All side faces are regular triangles, describe now. Which is the name of the solid when also the base is a regular triangle? – Many SDEs and the relations between them will be in the center of the discussions.

References

ANDZANS, A., MEISSNER, H. (Eds.): Creativity in Mathematics Education and the Education of Gifted Students: Proceedings of the International Conference. Riga, Latvia 2002

BAUERSFELD, H.: Subjektive Erfahrungsbereiche als Grundlage einer Interaktionstheorie des Mathematiklernens und -lehrens. In: Lernen und Lehren von Mathematik, Bd. 6, pp. 1-56, Aulis Verlag, Koeln, Germany 1983

BISHOP, A.: Visuelle Mathematik. In: Steiner, H.-G., Winkelmann, B. (Eds.): Fragen des Geometrieunterrichts (Untersuchungen zum Mathematikunterricht, IDM, 1), pp. 166-184, Aulis Verlag, Koeln, Germany 1981

GRAY, E. M., TALL, D. O.: Duality, Ambiguity and Flexibility in Successful Mathematical Thinking. In: Proceedings of PME-XV, vol. II, pp. 72-79, Assisi, Italy 1991

KIENEL, E.: Datenverarbeitung und die algorithmische Methode im Mathematikunterricht. Dissertation Muenster, Germany 1977

KIESSWETTER, K.: Modellierung von Problemloeseprozessen. In: Mathematikunterricht 29, Vol. 3, pp. 71-101, Friedrich Verlag, Seelze, Germany 1983

MEISSNER, H.: Cognitive Conflicts in Mathematics Learning. In: "European Journal of Psychology of Education", Vol. 1, No. 2/1986, p. 7-15, I.S.P.A. Lisboa, Portugal

MEISSNER, H.: Einstellung, Vorstellung, and Darstellung. In: Proceedings of the 26th International Conference for the Psychology of Mathematics Education, vol. I, pp. 156-161, Norwich, UK 2002

MEISSNER, H.: Constructing mathematical concepts with calculators or computers. In: Proceedings of CERME 3: Third Conference of the European Society for Research in Mathematics Education, Bellaria Italy 2003

MEISSNER, H., GRASSMANN, M., MUELLER- PHILIPP, S. (Eds.): Proceedings of the International Conference "Creativity and Mathematics Education". Westfaelische Wilhelms-Universitaet Muenster, Germany 1999

PEHKONEN, E.: Fostering of Mathematical Creativity. In: Zentralblatt fuer Didaktik der Mathematik, Vol. 3, pp. 63-67, Karlsruhe, Germany 1997

SHEFFIELD, L. J.: Extending the Challenge in Mathematics. Sage Publications Company. Thousand Oaks, California USA 2003

STRAUSS, S. (Ed.): U-shaped Behavioral Growth. Academic Press, New York 1982

UPITIS, R., PHILLIPS, E., HIGGINSON, W.: Creative Mathematics. Routledge, New York, USA 1997

VELIKOVA, E. (Ed.): Proceedings of the 3rd International Conference "Creativity in Mathematics Education and the Education of Gifted Students", Rousse Bulgaria 2003

ZIMMERMANN, B. et al. (Eds.): Kreatives Denken und Innovationen in mathematischen Wissenschaften. Tagungsband zum interdisziplinaeren Symposium an der Friedrich-Schiller-Universitaet Jena, Germany 1999

ZIZHAO, REN: Thoughts on Examination of Creativity in China's College Entrance Examination. Beijing, China (1999, unpublished)

see also
http://wwwmath.uni-muenster.de/didaktik/u/meissne/WWW/creativity.htm